Edge of a Half-Filled Landau Level

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We have investigated the electron occupation number of the edge of a quantum Hall (QH) droplet at $\nu = 1/2$ using exact diagonalization technique and composite fermion trial wavefunction. We find that the electron occupation numbers near the edge obey a scaling behavior. The scaling result indicates the existence of a well-defined edge corresponding to the radius of a compact droplet of uniform filling factor $1/2$. We find that the occupation number beyond this edge point is substantial, which is qualitatively different from the case of odd-denominator QH states. We relate these features to the different ways in which composite fermions occupy Landau levels for odd and even denominator states.

The fermion Chern-Simons theory of the half-filled Landau level has provided a simple and useful picture of the bulk state [1–3]. According to this theory effective composite particles move in the absence of external magnetic field and form a compressible metallic state. There is a strong current interest in the edge properties of this novel liquid [4–10]. However, unlike odd-denominator QH liquids, it is difficult to construct a gauge invariant chiral Luttinger theory [11,12] at $\nu = 1/2$ since the longitudinal conductivity is non-zero. Experimentally the electron tunneling I-V characteristic at the edge is found to behave as $I(V) \sim V^g$ with a ν-dependence of the tunneling exponent $g \approx \nu^{-1}$ [6]. Various theories of the one-particle spectral density in the edge state of compressible fractions exist [7–10]. There are some discrepancies in the prediction of the tunneling exponent among these theories.

Microscopically the fundamental properties of the edge are not well known in the case of compressible fractions such as $1/2$. Also it is not clear how their edge shapes compare with those of incompressible QH states. In the light of this, the electron occupation number n_k serves as a useful quantity to investigate since it can be compared directly with the well-known result for incompressible QH states at the filling factor $\nu = 1/M$: $n_k \propto (k - k_{ed})^{M-1}$, where the wavevector corresponding to the position of the edge is $k_{ed} = Mk_F$ and k_F is the Fermi wavevector [13,14].

An accurate calculation of the values of n_k near the edge requires a correct description of strong electron correlations using non-perturbative methods. In this paper we have investigated the properties of the compressible $\nu = 1/2$ edge using finite size exact diagonalization (ED) and finite-size scaling analysis, followed by a variational composite fermion wavefunction approach [15]. When a symmetric gauge is used the single particle states at large magnetic fields are characterized by the Landau level index n and angular momentum number m. For given n the possible values of angular mo-

mentum are $m = -n, -n+1, ..., 0, 1, ...$. We have computed the ground state distribution of the occupation number n_m on a disk for up to eleven particles (m is related to the wavevector k through $k = 2\pi m/R_m$, where $R_m = \sqrt{2(m+1)}$ is the mean radius of the angular momentum state m.) We set the magnetic length to one. We find: (a) The occupation number n_m of the edge satisfies a scaling relation so that the values of a big system may be extracted from those of relatively small systems. (b) The occupation number n_m is non-zero even for m greater than $m_{ed} = 2N - 1$. This is in sharp contrast to the N-particle incompressible states where n_m is non-zero only up to $m_{ed} = M(N-1)$. A variational wavefunction study also leads to the similar conclusion. (c) Numerically we find that n_m changes abruptly near $m = m_{eff}$, where n_m is very small beyond $m = m_{eff}$. (d) In the composite fermion picture, *some* of the composite fermions occupy higher (effective) Landau levels of composite fermions while the fractional Laughlin state is obtained when *all* of the composite fermions lie in the lowest Landau level. The tail region of n_m reflects this occupation behavior of composite fermions.

In the cleaved-edge construction used by Grayson *et al.* in their tunneling experiments, the edge potential is believed to be very sharp [6]. The basic physics of this situation may be modeled by fixing the total angular momentum of the droplet since for a certain range of the confinement strength the groundstate is given by the half-filled state with the total angular momentum $M_{TOT} = N(N-1)$ [16]. The maximum possible value of occupied angular momentum state may be determined as follows: First note that arbitrary states in the lowest Landau level may be written in the form

$$S[\{z_i\}] \prod_{i<j}(z_i - z_j) \exp(-\sum_i |z_i|^2/4), \qquad (1)$$

where $S[\{z_i\}]$ indicates an arbitrary *symmetric* polynomial [17]. Since the total angular momentum of a half-filled N-particle state is required to be $N(N-1)$ and

the Jastrow factor $\prod_{i<j}(z_i - z_j)$ carries the angular momentum $N(N-1)/2$, the remaining $N(N-1)/2$ units of angular momenta are carried by the symmetric polynomial. Therefore the angular momentum for a single particle state may be as large as the degree of the symmetric polynomial plus those from the Jastrow factor, which equals $m_c = (N-1) + N(N-1)/2 = (N+2)(N-1)/2$.

Figure 1 displays the ED results for n_m at various values of N. The oscillatory nature of the occupation numbers in the droplet had also been found for the incompressible fractions [13]. Explicit analytical result is available for $N = 3$ since the exact groundstate wavefunction is known for $(N, M_{TOT}) = (3, 6)$ [18]. The result of the ED agrees with high accuracy with this analytical result. The shape of the occupation number is rather different from that of $1/3$ state [13]. A striking feature of the ED result is that the occupation number exhibits a tail above k_{ed} (to be defined shortly) at $\nu = 1/2$. This property clearly departs from that of the incompressible edge where, for a sharp confining potential, the maximum occupation is fixed at $M(N-1)$. A sudden drop of occupation number occurs at k slightly greater than k_{ed} is observed for all our ED calculations, as illustrated in Tables I and II. For $N = 7$ to 10 particles, the drop in the occupation occurs when $m > m_{eff}$ with $m_{eff} = 15, 17, 20$, and 21 respectively. The ED results show that the effective cut-off radius m_{eff} is greater than m_{ed}, but still much smaller than the theoretical maximum value of m_c.

Figure 2 displays a scaling function that n_m of the edge obeys approximately

$$n_m = f((k - k_{ed})R_{ed}). \tag{2}$$

We have defined $m_{ed} = 2N - 1, R_{ed} = \sqrt{2(m_{ed}+1)}$, and $k_{ed} = 2\pi m_{ed}/R_{ed}$ respectively. The choice of our length scale R_{ed} is motivated by the fact that it is the radius of the compact droplet whose average filling factor is $1/2$. As we move to the edge some kind of scaling behavior is already apparent for the system sizes we have investigated. Since $k_{ed}R_{ed} = 2\pi(2N-1)$, the starting x-axis values in the figure are displaced horizontally by a fixed amount 4π for each increment of the particle number. While the peak positions of the occupation do not occur at the same point in the scaling variable we have chosen, the downward slide to zero occupation obviously follows a single curve. Finite-size corrections are visible in the increasing edge slope and the dwindling peak value for larger N. We expect that these corrections will ultimately vanish when we go to a large enough system. The occupation values seem to cross at $k = k_{ed}$. We have verified that the same data plotted as $f(k/k_{ed})$ also exhibit crossing at $k/k_{ed} \approx 1$. The presence of k_{ed} signifies the existence of a well-defined edge in the compresible states. The difference of R_{ed} and the cut-off radius where the occupation nearly vanishes, $W = R_{eff} - R_{ed}$ with $R_{eff} = \sqrt{2(m_{eff}+1)}$, is zero for the incompressible edges. From our numerical results we find that W

is nonzero in the thermodynamic limit for compressible edges.

Several key points of the ED results may be understood from trial variational wavefunction approach. These are: (a) $m_{eff} \neq m_c, m_e$ and grows with N, and (b) there is a sudden drop in n_m for $m > m_{eff}$. A good trial variational wavefunction for states close to half-filling is obtained by attaching two vortices to each electron:

$$\Psi = P_{LLL}[\prod_{i<j}(z_i - z_j)^2 \phi_{M^*}]. \tag{3}$$

The Slater determinant state ϕ_{M^*} consists of N Landau level states (n_i, m_i). For the half-filled state where the residual flux is zero, Rezayi and Read proposed to use the Slater determinant of free fermions for ϕ_{M^*} [2]. Recently Jain and Kamilla (JK) proposed an alternative way of writing the projected wavefunction in a symmetric gauge [15]. In our geometry it is more convenient to use JK's wavefunction. While JK's analysis focuses on the *bulk properties* of their variational wavefunction, here we focus on the edge properties of such states. As will become clear below, the variational state proves to be an excellent approximation to our ED calculation in many respects.

Since the total angular momentum of the groundstate at half-filling is $N(N-1)$, we obtain $M^* = \sum_{i=1}^{N} m_i = 0$. For $N = 7, 6, 5$, JK have chosen the projected groundstate with $M^* = 0$ as the compact states [4,1,1,1], [3,2,1], and [3,1,1] [15]. The agreement in the ground state energy computed in terms of JK's composite fermion wavefunction with the exact result is exceedingly good. Encouraged by this, we have computed the *occupation number* using JK's wavefunction and compared them with the exact results. We list the occupation numbers for $N = 5$ in Table II. For $N \geq 6$, the number of terms in the Slater determinant is too large to be handled exactly. As one can see in Table II, the agreement of numbers is excellent except at either ends of the distribution, where the relative difference is about 10 %. The sum of the occupation numbers in both cases is 5 to high accuracy. One can infer that JK's function is built out of a somewhat restricted set of states compared with the true ground state from the fact that small occupation probabilities at $m > m_{eff}$ are replaced by zeroes in JK's wavefunction. For generic N, the true ground state is likely to be a sum of several states of a given total angular momentum in which JK's state makes the most dominant contribution.

To estimate the last occupied state for general N, we need to know how to write down JK's wavefunction for arbitrary number of particles. For $N = 2q$ or $2q + 1$, we choose the trial wavefunctions as the following compact states

$$[q, 2, \overbrace{1, 1, ..., 1}^{q-2}]$$

$$[q+1,\underbrace{1,1,..,1}_{q}], \qquad (4)$$

where q is a positive integer. When other states are available with the same M_{TOT}, we must choose the one with the lowest energy. According to JK's analysis, the state which creates the least number of defects also has the lowest energy. By maximizing N_0, we can minimize the creation of the defects. Such consideration leads to the above states [19]. We will show that the above choice do correctly lead to some of the features observed numerically.

The total angular momentum of the first state is $M_{TOT} = 2q(2q - 1)$ and for the second state it is $M_{TOT} = 2q(2q + 1)$. Using the states described in Eq. (4), one can show that the last occupied single particle states occur at $m = 5q - 3$ and $5q$ for even and odd N respectively. This gives $m_{eff} = 15, 17, 20,$ and 22 for $N = 7 - 10$ in good comparison with the ED results $m_{eff} = 15, 17, 20,$ and 21. Using the values of $m_{eff} = 5q - 3$ and $5q$ we can estimate $W \propto \sqrt{N}$.

Our trial state, Eq. (4), shows that about half the composite fermions occupy the lowest Landau level, while the higher Landau levels are singly occupied by the other half. This is in contrast to the Laughlin states which, in JK's notation, is given by $N_0 = N$. That is, all of the composite fermions occupy the lowest Landau level to form an incompresible Laughlin state. Even in the case when Eq. (4) is not the groundstate, other possible states will also have a distribution of composite fermions in higher Landau levels. We believe such differences in the occupation behavior of composite fermions explain the observed difference of *electron occupation* for compressible/incompressible states.

In conclusion, our results indicate that the behavior of the edge at $\nu = 1/2$ is substantially different from that of the incompressible case. We find also that a sizable fraction of composite fermions occupy the higher Landau levels, which is reflected in the existence of a tail in n_m. It would be useful to calculate analytically the scaling function $f(x)$. Doing so would first require the construction of a proper effective theory of the edge which, like Wen's theory [11], will predict both the dynamic behavior (Green's function) and the ground-state behavior like the occupation number. It is hoped that our results may be useful in understanding such an effective low energy theory of the edges of compressible states.

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[15] J. K. Jain, and R. K. Kamilla, cond-mat/9704031; For a composite fermion occupying the n-th (effective) Landau level and the angular momentum m, the orbital wavefunction is given by

$$\eta_{n,m}^{CF}(z) = N_{n,m} e^{-|z|^2/4} z^{m+n} \partial^n \prod_k (z - z_k). \qquad (5)$$

The maximum angular momentum quantum number associated with z is $(N-1)+m$. The ground state is chosen as the compact state $[N_0, N_1, N_2, ...]$ with $N_0 \geq N_1 \geq N_2, ...$, where N_i is the number of composite fermions in the i-th (effective) Landau level.

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TABLE I. Occupation number for $N = 7$ obtained from exact diagonalization. Note that it drops suddenly at $m = 16$.

m	0	1	2	3	4	5	6	7	8
n_m	.3648	.3253	.4050	.6037	.7353	.7963	.8265	.6974	.6221
m	9	10	11	12	13	14	15	16	17
n_m	.5359	.3526	.2714	.2011	.1470	.0821	.0325	7.85×10^{-4}	2.71×10^{-4}

TABLE II. Comparison of occupation numbers for $N = 5$; ED vs. JK's trial wavefunction. Note that the ED result at $m = 11$ is 6.0×10^{-5} while that of the trial wavefunction is 0.

m	0	1	2	3	4	5
ED	.1805	.3671	.7375	.8442	.9012	.8035
JK	.2075	.3750	.7285	.8298	.8910	.7820
m	6	7	8	9	10	11
ED	.6208	.2549	.1562	.0967	.0373	6.0×10^{-5}
JK	.6076	.2608	.1692	.1069	.0419	0

FIG. 1. Occupation numbers for $N = 8(\circ), 9(\square), 10(\diamond), 11(\triangle)$. Extremely small occupation numbers at large $m > m_{eff}$ are not shown.

FIG. 2. Occupation numbers of Fig. 1 in a scaling form $n_m = f((k - k_{ed})R_{ed})$. The quantities k_{ed} and R_{ed} are defined in the text.